Local Fact Change Logic, Memory Logic and Expressive Power

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Talk Overview

Boolean (network) games

Local Fact Change

Undecidability via Memory Logic

Measuring expressive power

Conclusion

Boolean Games

- We have a set Prop of propositions.
- Each player controls a subset of Prop.
- Each player s has a formula γ_s of propositional logic as their *goal*.
- By choosing the valuation on their propositions, s tries to make γ_s true.

Boolean Network Games

- Players are arranged in a network.
- Each player controls *all the propositions at their position*.
- Each player s has a formula γ_s of *modal* logic as their goal.
- By choosing the valuation *at their position*, s tries to make γ_s true.

BNGs: Strategies and equilibria

Definition (Strategy (profile))

A *strategy* is a subset of Prop. A strategy profile is a function $V : W \to 2^{\mathrm{Prop}}$.

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A strategy profile V is a *Nash equilibrium* if there is no player who can do better by changing strategy.

How can we make this definition more precise? We need a logical way to talk about *changing strategies*.

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$\mathfrak{F}, V, s \models (\varphi \wedge \psi)$	iff	$\mathfrak{F}, V, s \models \varphi$ and $\mathfrak{F}, V, s \models \psi$
$\mathfrak{F}, V, s \models \Diamond\varphi$	iff	$\mathfrak{F}, V, t \models \varphi$ for some t with Rst

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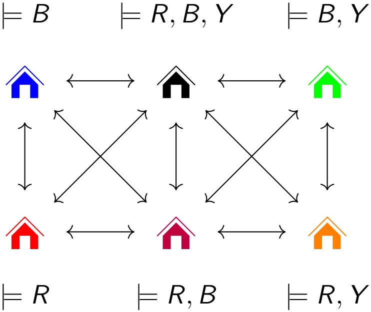
$\mathfrak{F}, V, s \models \Diamond\varphi$	iff	$\mathfrak{F}, V, t \models \varphi$ for some t with Rst
$\mathfrak{F}, V, s \models \bigcirc\varphi$	iff	$\mathfrak{F}, V^s_A, s \models \varphi$ for some $A \subseteq$ Prop

$\bigcirc$ changes the valuation *but only at the current state*.

Example



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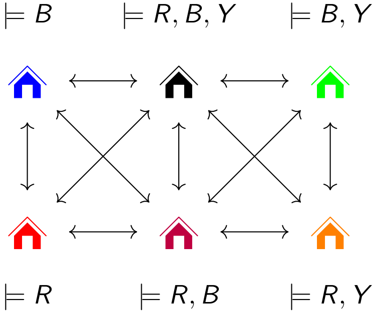

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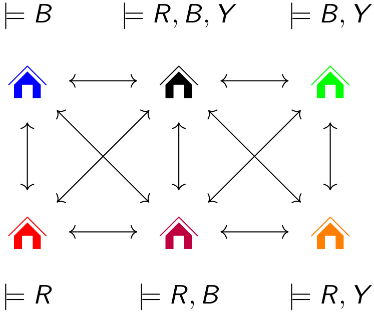

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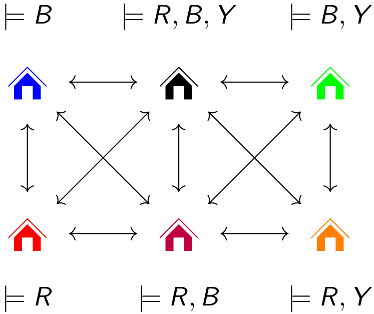

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- V is a Nash equilibrium iff $\mathfrak{F}, V, s \models \bigcirc\gamma_s \rightarrow \gamma_s$ for every player s.

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- LFC is strictly more expressive than basic modal logic: $\bigcirc \Diamond p \to \Diamond p$ is valid on a frame iff it is irreflexive.
- How expressive is it?

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Theorem

The satisfiability problem for memory logic is undecidable (Mera 2009).

Undecidability of LFC

Main idea

We translate satisfiability problems for M to satisfiability problems for LFC.

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The satisfiability problem for LFC is undecidable.

There are many more details. In particular, *the translation is not direct.* How do M and LFC compare?

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Defining expressive power comparisons

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Definition ($\leq$, "translation")

A $\leq$ B if there is a $\mathfrak{T} : \mathcal{L}_{\mathsf{A}} \to \mathcal{L}_{\mathsf{B}}$ such that for all models $\mathfrak{M}$,

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Definition ($\preceq$, "no more distinctions")

A $\preceq$ B if for every pair of models $\mathfrak{M}_1, \mathfrak{M}_2$, if there is $\varphi \in \mathcal{L}_\mathsf{A}$ such that $\mathfrak{M}_1 \models_\mathsf{A} \varphi$ and $\mathfrak{M}_2 \not\models_\mathsf{A} \varphi$ then there is $\psi \in \mathcal{L}_\mathsf{B}$ such that $\mathfrak{M}_1 \models_\mathsf{B} \psi$ and $\mathfrak{M}_2 \not\models_\mathsf{B} \psi$.

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Fact

If $\mathsf{A} \leq \mathsf{B}$ *then* $\mathsf{A} \preceq \mathsf{B}$.

Proof.

Take $\mathfrak{T}(\varphi)$ *for* ψ. □

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- A $\not\preceq$ B iff there is a pair of models $\mathfrak{M}_1, \mathfrak{M}_2$ that A can distinguish that B cannot.
- To compare M and LFC, we need a modal invariance notion.
- When are two models indistinguishable for LFC?

Ehrenfeucht-Fraïssé Games for LFC

$EF(\mathfrak{F}_1, \mathfrak{F}_2, V_1, V_2, s_1, s_2)$

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  2. The following occur in order:
     - 2.1 Spoiler chooses $i \in \{1, 2\}$ (j is the other)
     - 2.2 Spoiler chooses $t_i \in W_i$ such that $R_i s_i t_i$.
     - 2.3 If there is no t_j with $R_j s_j t_j$, Spoiler wins. Otherwise, Duplicator picks such a t_j. We play $EF(\mathfrak{F}_1, \mathfrak{F}_2, V_1, V_2, t_1, t_2)$.

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In an infinite game, Duplicator wins.

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Fact

If Duplicator has a winning strategy then for all φ,

$$\mathfrak{F}_1, V_1, s_1 \models_{\mathrm{LFC}} \varphi \quad \textit{iff} \quad \mathfrak{F}_2, V_2, s_2 \models_{\mathrm{LFC}} \varphi.$$

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$\mathfrak{G}_1$

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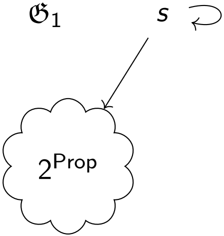

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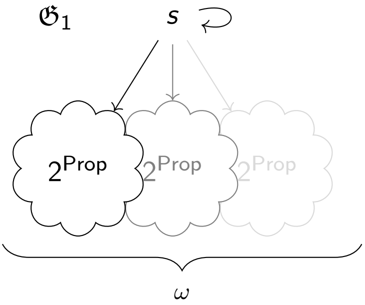

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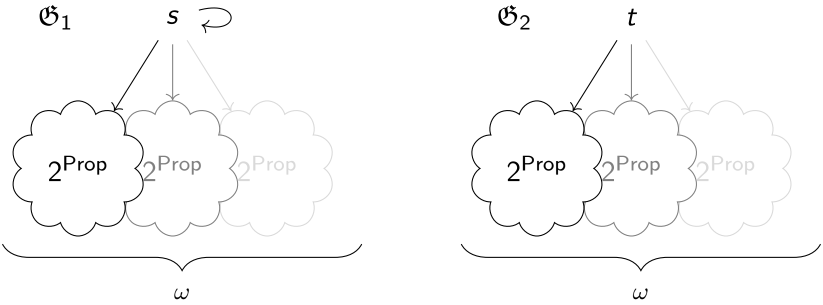

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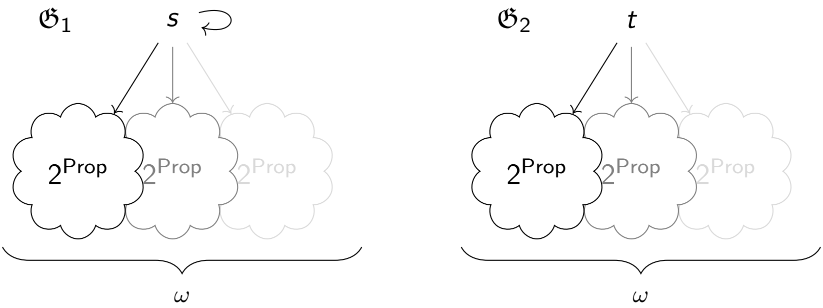
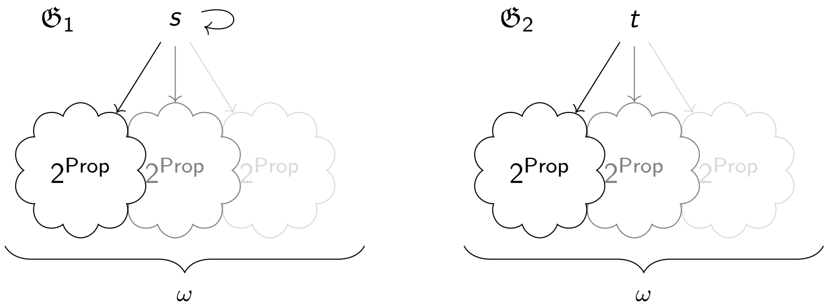


$\mathfrak{G}_1, \emptyset, s \models_{\mathsf{M}} ⓡ \Diamond ⓚ$ $\qquad$ $\mathfrak{G}_2, \emptyset, t \not\models_{\mathsf{M}} ⓡ \Diamond ⓚ$

M $\not\leq$ LFC (cont.)



Duplicator has a winning strategy in LFC

- s and t have the same valuation.
- Every node has a neighbour.
- For every node spoiler picks, there is an *unvisited* node with the same valuation.

M $\not\leq$ LFC (cont.)

- $\mathfrak{G}_1, \emptyset, s \models_{\mathsf{M}}$ ⓡ $\Diamond$ ⓚ and $\mathfrak{G}_2, \emptyset, t \not\models_{\mathsf{M}}$ ⓡ $\Diamond$ ⓚ

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- M $\not\succeq$ LFC
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$\mathsf{M} \not\leq \mathsf{LFC}$ (cont.)

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What about finite models?

Restricted EF Games for LFC

$EF_R(\mathfrak{F}_1, \mathfrak{F}_2, V_1, V_2, s_1, s_2)$

- *If* $V_1(s_1) \neq V_2(s_2)$ then Spoiler wins.
- *Else* if both s_1 and s_2 have no neighbours then Duplicator wins.
- *Else* Spoiler chooses one of the following two moves:
  1. Spoiler picks $A \subseteq \text{Prop}$. We play $EF(\mathfrak{F}_1, \mathfrak{F}_2, {V_1}^{s_1}_A, {V_2}^{s_2}_A, s_1, s_2)$.
  2. The following occur in order:
     - 2.1 Spoiler chooses $i \in \{1, 2\}$ (j is the other)
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In an infinite game, Duplicator wins.

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Fact

If Duplicator has a winning strategy, $\mathrm{MD}(\varphi) \leq n$ *and* $\mathrm{At}(\varphi) \subseteq B$,

$$\mathfrak{F}_1, V_1, s_1 \models_{\mathsf{LFC}} \varphi \quad \textit{iff} \quad \mathfrak{F}_2, V_2, s_2 \models_{\mathsf{LFC}} \varphi.$$

M $\not\leq$ LFC on finite models

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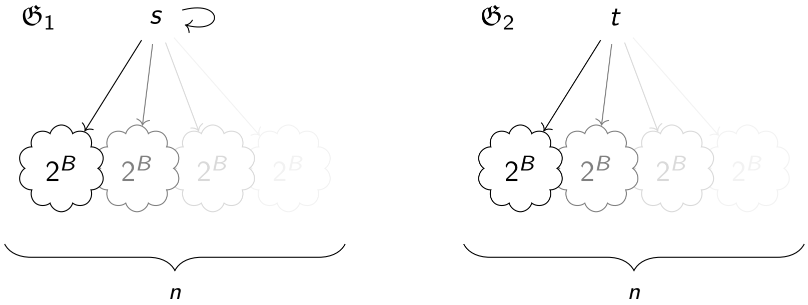

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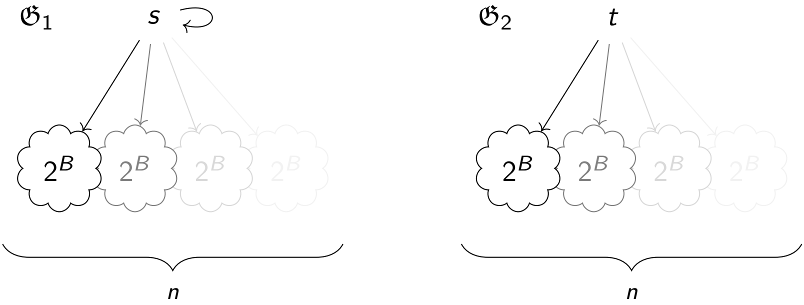

- Duplicator has the same winning strategy as before in $EF_R(\mathfrak{G}_1, \mathfrak{G}_2, V_1, V_2, s, t, B, n)$.

M $\not\leq$ LFC on finite models (cont.)

- For any translation T, we can construct a pair $\mathfrak{G}_1, \mathfrak{G}_2$ such that
  1. $\mathfrak{G}_1, \emptyset, s \models_{\mathsf{M}} ⓡ \Diamond ⓚ$
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- So no translation satisfies the definition of $\leq$.

Definition ($\leq$)

A $\leq$ B if there is a $\mathfrak{T} : \mathcal{L}_{\mathsf{A}} \to \mathcal{L}_{\mathsf{B}}$ such that for all models $\mathfrak{M}$,

$$\mathfrak{M} \models_{\mathsf{A}} \varphi \quad \text{iff} \quad \mathfrak{M} \models_{\mathsf{B}} \mathfrak{T}(\varphi).$$

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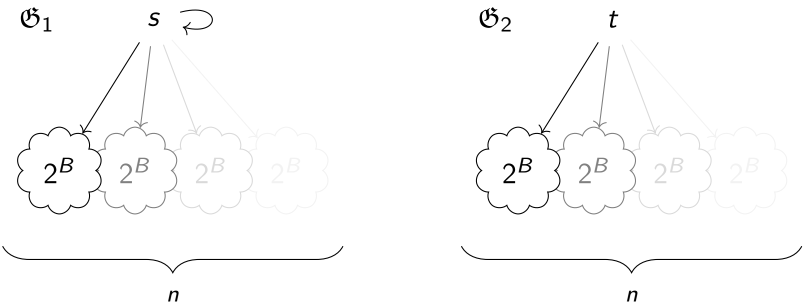
BUT!

Fix n and $B \subsetneq \mathrm{Prop}$. Take $q \notin B$.



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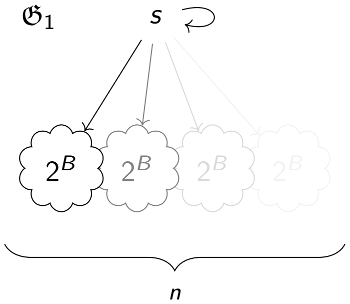

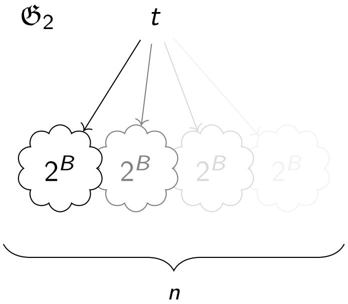

$\mathfrak{G}_1, s \models_{\text{LFC}} \bigcirc \Diamond q$ $\mathfrak{G}_2, t \not\models_{\text{LFC}} \bigcirc \Diamond q$

- So LFC can distinguish all our countermodels.

What do EF Games for LFC correspond to?

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Definition (Strongly connected component)

Let $\mathfrak{F} = (W, R)$ and $W \in A$. $SCC(s)$ is the smallest subgraph $\mathfrak{G} = (W', R')$ of $\mathfrak{F}$ such that if there is a path in $\mathfrak{F}$ from s to t, and from t to s, then $t \in W'$.

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Definition (Isobisimulation)

Let $\mathfrak{M}_1 = (W_1, R_1, V_1)$ and $\mathfrak{M}_2 = (W_2, R_2, V_2)$. A relation $Z \subseteq W_1 \times W_2$ is an *isobisimulation* if the following clauses hold:

- **Non-empty** $Z \neq \emptyset$
- **Agree** If $s_1 Z s_2$ then $V_1(s_1) = V_2(s_2)$.
- **Zig** If $s_1 Z s_2$ and $R_1 s_1 t_1$ then there is t_2 with $R_2 s_2 t_2$.
- **Zag** If $s_1 Z s_2$ and $R_2 s_2 t_2$ then there is t_1 with $R_1 s_1 t_1$.
- **Isomorphism** If $s_1 Z s_2$ then there is an isomorphism $f : SCC(s_1) \rightarrow SCC(s_2)$ such that $f(s_1) = s_2$.

Isobisimulation and LFC

Theorem

For finite pointed models $\mathfrak{M}_1, s_1$ and $\mathfrak{M}_2, s_2$, the following are equivalent:

1. *There is an isobisimulation Z with $s_1 Z s_2$.*
2. *Duplicator has a winning strategy in $EF(\mathfrak{F}_1, \mathfrak{F}_2, V_1, V_2, s_1, s_2)$.*
3. *For every $\varphi \in \mathcal{L}_{LFC}$ we have $\mathfrak{M}_1, s_1 \models \varphi$ iff $\mathfrak{M}_2, s_2 \models \varphi$.*

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Proof idea.

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- $2 \Rightarrow 3$ *was mentioned above.*

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- $1 \Rightarrow 2$*: Duplicator follows the isobisimulation. Spoiler's valuation-change move only affects repeat visits - when we're guaranteed to be in an isomorphic component.*
- $2 \Rightarrow 1$*: Use Duplicator's strategy to build an isobisimulation. Key idea: Spoiler can label all the vertices of a SCC.*

M $\preceq$ LFC on finite models

Theorem

Let $\mathfrak{M}_1, s_1$ and $\mathfrak{M}_2, s_2$ be finite pointed models, and let Z be an isobisimulation with $s_1 Z s_2$. Then for all $\varphi \in \mathcal{L}_{\mathsf{M}}$,

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Proof.

Similar to that for LFC. *Duplicator's strategy is just to follow the isobisimulation.* □

$\mathsf{M} \preceq \mathsf{LFC}$ on finite models

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Corollary

$\mathsf{M} \preceq \mathsf{LFC}$ *on finite models.*

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Is M *more* expressive than LFC?

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LFC $\not\leq$ M *for finite models.*

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LFC $\not\preceq$ M *for finite models.*

Proof.

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$$\text{LFC} \overset{?}{\preceq} \text{M}$$

Open questions

Expressive Power

- What is the relationship between M and isobisimulation?
- Is LFC $\preceq$ M?
- What other situations do $\leq$ and $\preceq$ give different judgements?
- What is the relationship between other logics (e.g. Hybrid logic) and isobisimulation?
- Restricted tree model property? Decidability for classes of models?

General

- What weakenings of LFC will make it decidable?
- What is the exact relationship between LFC and Nash equilibria for BNGs? Can we say the logic of Nash equilibria is undecidable?

Thank you

Related Work

- Propositional formulae, with agents controlling a subset of the atomic variables: Coalition Logics of Propositional Control (van der Hoek and Wooldridge 2005), Boolean Games (Harrenstein et al. 2001).

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- PDL with local and global assignments to propositional variables: PDL+GLA (Tiomkin and Makowsky 1985)

Areces, Carlos et al. (June 2011). "The Expressive Power of Memory Logics". In: *The Review of Symbolic Logic* 4.02, pp. 290–318.

Balbiani, Philippe, Andreas Herzig, and Nicolas Troquard (2013). "Dynamic Logic of Propositional Assignments: A Well-Behaved Variant of PDL". In: *Proceedings of the 2013 28th Annual ACM/IEEE Symposium on Logic in Computer Science*. LICS '13. Washington, DC, USA: IEEE Computer Society, pp. 143–152. ISBN: 978-0-7695-5020-6.

Harrenstein, Paul et al. (2001). "Boolean Games". In: *Proceedings of the 8th Conference on Theoretical Aspects of Rationality and Knowledge*. Morgan Kaufmann Publishers Inc., pp. 287–298.

Mera, Sergio Fernando (2009). "Modal Memory Logics". PhD Thesis. Buenos Aires: Universidad de Buenos Aires. 165 pp.

Tiomkin, M. L. and J. A. Makowsky (Jan. 1, 1985). "Propositional Dynamic Logic with Local Assignment". In: *Theoretical Computer Science* 36, pp. 71–87.

Van der Hoek, Wiebe and Michael Wooldridge (May 2005). "On the Logic of Cooperation and Propositional Control". In: *Artificial Intelligence* 164.1–2, pp. 81–119.

$$\models \Box^n p \leftrightarrow (\bullet \Box^n p \vee (\bullet(p \leftrightarrow \Box^n p) \wedge p)))$$